Name: Key Date: ________

Pre-Calculus

Practice Test, L. 4.5 to 4.7

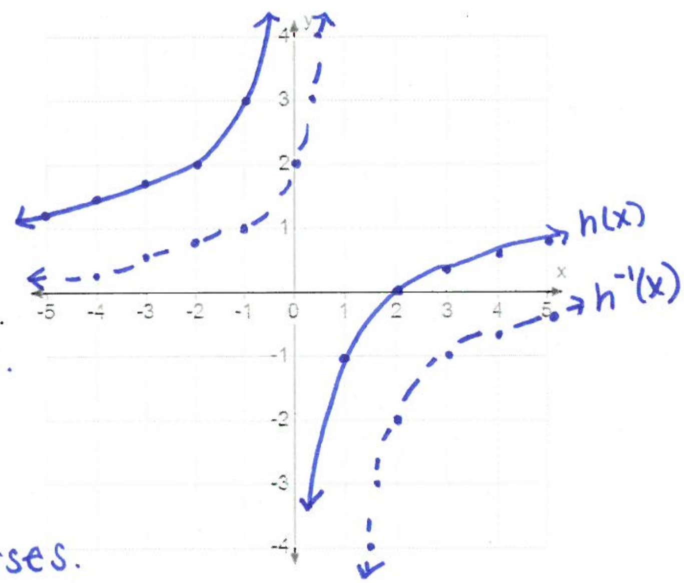
(Your calculator may be used as a tool, but all results must also be proved algebraically!)

$h(x)$

x	y
-5	1.4
-4	1½
-3	1 2/3
-2	2
-1	3
0	und.
1	-1
2	0
3	1/3
4	1/2
5	.6

For #1 – 3, let $h(x)=1-\frac{2}{x}$.

1. Sketch the graphs of $h(x)$ and $h^{-1}(x)$ at the right. Show $h^{-1}(x)$ as a dashed line.



2. Find a rule for $h^{-1}(x)$.

$y = 1-\frac{2}{x}$ $\quad x = 1-\frac{2}{y}$ $\quad x-1 = \frac{-2}{y}$

$y(x-1) = -2$

$h^{-1}(x) = \frac{-2}{x-1}$ $\quad y = \frac{-2}{(x-1)}$

3. Show algebraically that $h(x)$ and $h^{-1}(x)$ are inverse functions.

$h(h^{-1}(x)) = 1-\frac{2}{\left(\frac{-2}{x-1}\right)} = 1 - \cancel{2}\left(\frac{x-1}{\cancel{-2}}\right) = 1+x-1 = x.$

$h^{-1}(h(x)) = \frac{-2}{\left(1-\frac{2}{x}\right)-1} = \frac{-2}{\frac{-2}{x}} = \cancel{-2}\cdot\frac{x}{\cancel{-2}} = x.$

Thus, h(x) and $h^{-1}(x)$ are inverses.

For #4 – 6, use the information given below.

Pepsi is researching the most cost effective way to make pop cans. Each can costs 3¢ per ~~cubic~~ square centimeter for the bottom and top of the can and 2¢ per ~~cubic~~ square centimeter for the sides.

4. Express the cost of making one can as a function of the radius r and height h of the can.

$S.A. = \underbrace{2\pi r^2}_{\text{top/bottom}} + \underbrace{2\pi r h}_{\text{sides}}$

$C(r,h) = 3(2\pi r^2)+2(2\pi r h) = 6\pi r^2 + 4\pi r h$

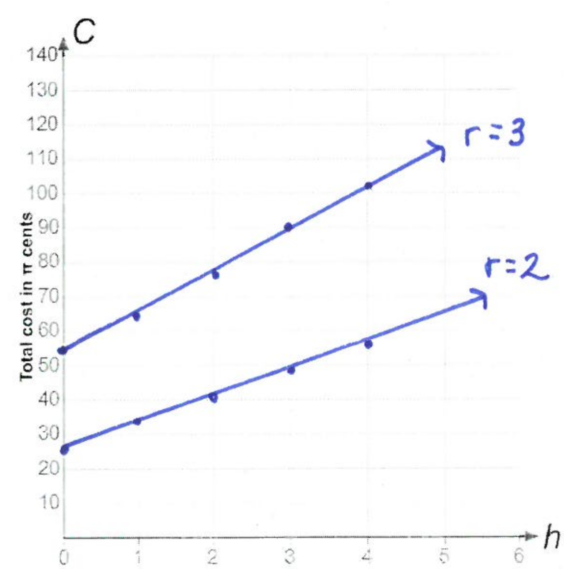
5. Draw two curves of constant radius $r(h,C)$ in an rh-plane using $r=2$ cm and $r=3$ cm.

r=2

h	C
0	24π
1	32π
2	40π
3	48π
4	56π

r=3

h	C
0	54π
1	66π
2	78π
3	90π
4	102π



6. If the volume of a can is 108π cm^3, rewrite the cost formula in terms of radius only.

$V = \pi r^2 h$

$\frac{108\pi}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$

$h = \frac{108}{r^2}$

$C(r) = 6\pi r^2 + 4\pi r\left(\frac{108}{r^2}\right)$

$C(r) = 6\pi r^2 + \frac{432\pi}{r}$

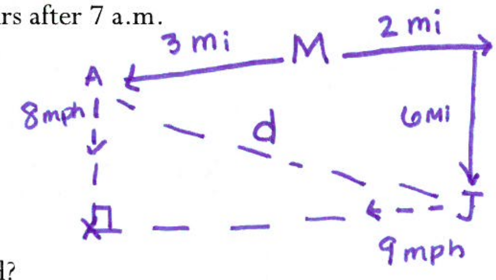
Pre-Calculus

(Your calculator may be used as a tool, but all results must also be proved algebraically!)

For #7 – 9, use the information given below.

At 7 a.m., Andrew is 3 miles west of Minster heading south at 8 mph on S.R. 364. At the same time, Jon is 2 miles east and 6 miles south of Minster heading west on Cardo road at 9 mph.

7. Express the distance d between Andrew and Jon as a function of time t hours after 7 a.m.



$d^2 = (6-8t)^2 + (5-9t)^2$

$d = \sqrt{(6-8t)^2 + (5-9t)^2}$

$d = \sqrt{145t^2 - 186t + 61}$

8. Will Jon reach State Route 364 before Andrew passes south of Cardo Road?

Jon: $d = rt$
$5 = 9t$
$\frac{5}{9} = t$

Andrew: $d = rt$
$6 = 8t$
$3/4 = t$

Yes. Jon will reach 364 before Andrew passes south of Cardo Road.

9. How far apart will Jon and Andrew be at 7:30 a.m.?

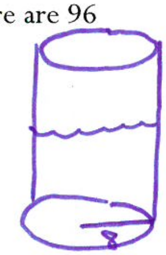
7:30 AM ⇒ $t = 1/2$ hr.

$d = \sqrt{145(1/2)^2 - 186(1/2) + 61}$

$d = \sqrt{4.25}$

$d \approx 2.062$ mi.

10. A cylindrical tank 16 ft in diameter fills with water at the rate of 12 ft^3/sec. Assuming that there are 96 ft^3 of water in the tank at time $t = 0$, express the depth of the water as a function of time.



$r = 8$ ft

$\Delta V = 12$ ft^3/sec

Initial $V = 96$ ft^3/sec

$12 = \pi(8)^2 h$

$96 = \pi(8)^2 h$

initial $h = \frac{3}{2\pi} = h$

Depth $= \frac{3}{16\pi} t + \frac{3}{2\pi}$

$\Delta h = \frac{3}{16\pi} = h$

For #11 – 13, use the information given below.

Rectangle $ABCD$ has two vertices on the semicircle $y = \sqrt{16 - x^2}$ and two vertices on the x-axis as shown at the right.

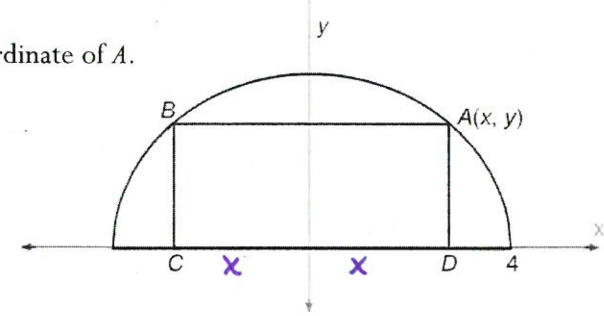
$A = w \cdot h$

11. Express the area of the rectangle as a function of the x-coordinate of A.

$w = 2x$

height $= \sqrt{16 - x^2}$

$A = 2x\sqrt{16 - x^2}$



12. What is the domain of the area function?

$0 < x < 4$

13. Use a graphing calculator to find the maximum area.

$x \approx 2.828 = 2\sqrt{2}$

$A \approx 16$ units^2