Name: Key Date: ________

Pre-Calculus

Practice Quiz, L. 20.1 (Beginning of Derivatives)

1. Using the limit definition of slope, find the derivative of the curve $f(x) = -3x^2 - 1$ and then find the slope at $x = 4$.

$$\lim_{h \to 0} \frac{-3(x+h)^2 - 1 - (-3x^2 - 1)}{h} = \lim_{h \to 0} \frac{\cancel{-3x^2} - 6xh - 3h^2 \cancel{-1} \cancel{+3x^2} \cancel{+1}}{h}$$

$$\lim_{h \to 0} \frac{-6xh - 3h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(-6x - 3h)}{\cancel{h}} = \lim_{h \to 0} -6x - 3h = -6x \quad \boxed{f'(4) = -6(4) = -24}$$

$$\boxed{f'(x) = -6x}$$

2. Using the limit definition of slope, find the derivative of the curve $f(x) = x^3 + 2x$.

$$\lim_{h \to 0} \frac{(x+h)^3 + 2(x+h) - (x^3 + 2x)}{h}$$

$$\lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3h^2x + h^3 + \cancel{2x} + 2h \cancel{-x^3} \cancel{-2x}}{h} = \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3hx + h^2 + 2)}{\cancel{h}} = \lim_{h \to 0} 3x^2 + 3hx + h^2 + 2$$

$$\boxed{f'(x) = 3x^2 + 2}$$

3. Write the equation of the tangent line to $g(x) = -4x^3 + 6x^2 - 3$ at $x = -1$.

$g'(x) = -12x^2 + 12x \quad g'(-1) = -12(-1)^2 + 12(-1) = -24 \quad (-1, 7)$

$g(-1) = -4(-1)^3 + 6(-1)^2 - 3 = 7 \quad 7 = -24(-1) + b$

$7 = 24 + b$

$-24 \quad -24$

$-17 = b$

$$\boxed{\text{Tangent Line at } x = -1:\ y = -24x - 17}$$

4. Write out, but do not evaluate, an expression using limits to find the slope of $h(x) = \sqrt{x^2 - 4}$.

$$\boxed{h'(x) = \lim_{h \to 0} \frac{\sqrt{(x+h)^2 - 4} - \sqrt{x^2 - 4}}{h}}$$

For each function, find $\frac{dy}{dx}$.

5. $y = 3x^3 + \pi x^2 + 16x$

$$\boxed{\frac{dy}{dx} = 9x^2 + 2\pi x + 16}$$

6. $y = \frac{2}{3x^2}$

$$\frac{dy}{dx} = \frac{(3x^2)(0) - 2(6x)}{(3x^2)^2} = \frac{-12x}{9x^4} = \frac{-4}{3x^3}$$

$$\boxed{\frac{dy}{dx} = \frac{-4}{3x^3}}$$

7. $y = (2x^5 + 5x)\left(7 - \frac{2}{3}x^3\right)$

$$\frac{dy}{dx} = \left(7 - \frac{2}{3}x^3\right)(10x^4 + 5) + (2x^5 + 5x)(-2x^2)$$

$$\frac{dy}{dx} = 70x^4 + 35 - \frac{20}{3}x^7 - \frac{10}{3}x^3 - 4x^7 - 10x^3$$

$$\boxed{\frac{dy}{dx} = -\frac{32}{3}x^7 + 70x^4 - \frac{40}{3}x^3 + 35}$$

8. $y = \frac{x^4 - x + 12}{5x - 3}$

$$\frac{dy}{dx} = \frac{(5x-3)(4x^3 - 1) - (x^4 - x + 12)(5)}{(5x-3)^2}$$

$$\frac{dy}{dx} = \frac{\cancel{20x^4} - \cancel{12x^3} - \cancel{5x} + \cancel{3} - \cancel{5x^4} + \cancel{5x} - \cancel{60}}{(5x-3)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{15x^4 - 12x^3 - 57}{(5x-3)^2}}$$

9. Find the derivative of the curve $f(x)=\frac{-3}{2x+1}$.

$$f'(x)=\frac{(2x+1)(0)-(-3)(2)}{(2x+1)^2}$$

$$\boxed{f'(x)=\frac{6}{(2x+1)^2}}$$

10. Find the x- and y-intercepts of the line that is tangent to the curve $y=x^3$ at the point $(-2,-8)$.

$y'=3x^2 \quad y'(-2)=3(-2)^2=12 \quad y=(-2)^3=-8$

$-8=12(-2)+b$
$-8=-24+b$
$+24 \quad +24$
$16=b$

$0=12x+16$
$-16=12x$
$\frac{-16}{12}=\frac{12x}{12}$
$\frac{-4}{3}=x$

$$\boxed{\begin{array}{l}\text{x-intercept: } \frac{-4}{3} \text{ or } \left(\frac{-4}{3},0\right)\\ \text{y-intercept: } 16 \text{ or } (0,16)\end{array}}$$

11. Find y''' for $y=x^3-\frac{3}{x}$.

$y=x^3-3x^{-1}$

$y'=3x^2+3x^{-2}$
$y''=6x-6x^{-3}$
$y'''=6+18x^{-4}$

$$\boxed{y'''=6+\frac{18}{x^4}}$$

12. Find $\frac{d}{dx}\left(v(x)\left(2x^2-4x\right)\right)$ when $x=2$ given that $v(2)=7$ and $v'(2)=-1$.

$\frac{d}{dx}(v(x)(2x^2-4x))=(2x^2-4x)(-1)+7(4x-4)$
$=[2(2)^2-4(2)](-1)+7(4(2)-4)$
$=0+7(4)$

$$\boxed{\frac{d}{dx}(v(x)(2x^2-4x))=28}$$

13. Find the horizontal tangents to the curve $y=x^4-4x^2+1$.

$\frac{dy}{dx}=4x^3-8x$

$4x^3-8x=0$
$4x(x^2-2)=0$
$\boxed{x=0} \quad x^2-2=0$
$x^2=2$
$\boxed{x=\pm\sqrt{2}}$

$x=0, \ y=1$
$x=\sqrt{2}, \ y=-3$
$x=-\sqrt{2} \quad y=-3$

$$\boxed{\begin{array}{l}\text{Horizontal Tangent Lines:}\\ y=1\\ y=-3\end{array}}$$