Name: ______________________ Date: ________

Pre-Calculus
2nd Semester Exam Review

1. Rewrite $\ln 2 = 0.693$ using exponential notation.

$e^{0.693} = 2$

2. The common logarithm has a base of ___ten___

3. Express $\log_b N$ in terms of natural logs.

Change of base: $\log_b N = \frac{\log N}{\log b} = \frac{\ln N}{\ln b}$

4. Simplify $\frac{2^6(2^{-3})}{2^4}$.

$\frac{2^3}{2^4} = \frac{1}{2}$

5. Simplify $(4^{-1} - 2^{-3})^{-2}$.

$\left(\frac{1}{4} - \frac{1}{8}\right)^{-2} = \left(\frac{1}{8}\right)^{-2} = (8)^2 = 64$

6. Simplify $\frac{5n^{4/5} - 20n^{-2/5}}{10n^{3/5}}$

$\frac{5n^{4/5}}{10n^{3/5}} - \frac{20n^{-2/5}}{10n^{3/5}} = \frac{1}{2}n^{1/5} - 2n^{-1} = \frac{\sqrt[5]{n}}{2} - \frac{2}{n}$

7. Simplify $\frac{\sqrt[3]{c^4}}{\sqrt{c^7}}$.

$\frac{c^{4/3}}{c^{7/2}} = c^{4/3 - 7/2} = c^{\frac{8}{6} - \frac{21}{6}} = c^{-13/6} = \frac{1}{c^{13/6}} = \frac{1}{\sqrt[6]{c^{13}}}$

$= \frac{1}{c^2\sqrt[6]{c}}$

8. Solve $(2x)^{-3} = 27$.

$\frac{1}{(2x)^3} = \frac{27}{1}$

$(2x)^3(27) = 1$ $\quad x = \frac{1}{6}$

$8x^3 \cdot 27 = 1$

$x^3 = 1/216$

9. Solve $\frac{4^{x+1}}{2^3}=32$.

$\frac{(2^2)^{x+1}}{2^3}=2^5$ $\quad \frac{2^{2x+2}}{2^3}=2^5$ $\quad 2^{2x-1}=2^5$

$2x-1=5$ $\quad 2x=6$

$x=3$

10. Solve $\log_4(3x)=3$.

$4^3=3x$ $\quad x=\frac{64}{3}=21\frac{1}{3}$

$64=3x$

11. Compute $\log_2 10$ to the nearest hundredth.

$\frac{\log 10}{\log 2}\approx 3.32$

12. What is the y-intercept of the graph of $y=-2(4)^x$?

$(0,-2)$

*A radioactive element has a half-life of 6 days. If 200 grams are now present in a laboratory:

13. Write an equation for the amount present after n days.

$A(n)=200\left(\frac{1}{2}\right)^{n/6}$

14. How much will remain after 21 days?

$A(21)=200\left(\frac{1}{2}\right)^{21/6}$

$A(21)\approx 17.68$ grams

15. Mr. Lee invests $3000 at an annual interest rate of 4%, compounded monthly. What will the balance in his account be after 2 years and 5 months?

$A(t)=3000\left(1+\frac{0.04}{12}\right)^{12t}$

$A(29/12)=3000\left(1+\frac{0.04}{12}\right)^{12(29/12)}$

$\approx \$3,303.95$

*If $\log M=x$ and $\log N=y$, express each of the following in terms of x and y.

16. $\log\left(M^2N\right)^2$

Law of Logs

$2\log(M^2N)$

$2\log M^2+2\log N$

$4\log M+2\log N$

$4x+2y$

17. $\log(200M)$

$\log 200+\log M$

$\log 2+\log 100+\log M$

$\log 2+2+x$

18. Solve $\log_2(2x)+\log_2 x = 18$.

Laws of Logs

$\log_2 2x^2 = 18$ $\quad 2^{18} = 2x^2$ $\quad 2^{17} = x^2$ $\quad x = 2^{17/2} \approx 362.04$

19. Express y as a function of x for $3\ln y = 7 - \ln x$.

Solve for y

$3\ln y + \ln x = 7$

$\ln y^3 + \ln x = 7$

$\ln y^3 x = 7$ $\quad e^7 = y^3 x$

$y^3 = \frac{e^7}{x}$ $\quad y = \sqrt[3]{\frac{e^7}{x}}$

20. Find the exact value of $\log 5^{\log_5 100}$.

$\log 100 = \boxed{2}$

21. A sequence is called a geometric sequence if the ratio of any two consecutive terms is constant.

22. Find the next term in the sequence $15, 21, 27, \ldots$

$+6 \quad +6$

$27+6 = \boxed{33}$

23. Write an explicit formula form for the sequence $16, 40, 100, \ldots$

$\times 2.5 \quad \times 2.5$

geometric

$t_n = t_1 r^{n-1}$

$t_n = 16(2.5)^{n-1}$ or $t_n = 16(5/2)^{n-1}$

24. What is the fifth term in the sequence defined recursively by: $t_1 = 4, t_n = t_{n-1} + 2$.

$t_1 = 4 \quad t_2 = 6 \quad t_3 = 8 \quad t_4 = 10$

$\boxed{t_5 = 12}$

25. Write a recursive formula for the sequence $16, 4, 1, \frac{1}{4}, \ldots$

$\times \frac{1}{4} \quad \times \frac{1}{4}$

geometric

$t_1 = 16$

$t_n = \frac{1}{4} t_{n-1}$

26. The third term of an arithmetic sequence is 282 and the sixth term is 254. Find t_{27}.

$t_3 = 282 \quad t_6 = 254$

$\underline{\ \ }_{t_1} \ \underline{\ \ }_{t_2} \ \underline{282}_{t_3} \ \underline{\ \ }_{t_4} \ \underline{\ \ }_{t_5} \ \underline{254}_{t_6}$

$282 = t_1 + -28/3(2)$

$282 = t_1 - 56/3$

$\frac{902}{3} = t_1$

$d = \frac{254-282}{3}$

$d = -28/3$

$t_n = \frac{902}{3} - \frac{28}{3}(n-1)$

$t_{27} = \frac{902}{3} - \frac{28}{3}(26) = \boxed{58}$

27. In a geometric sequence, the first term is 5, and the 5^{th} term is 1280. Find the common ratio.

$t_1 = 5$ $\quad t_5 = 1280$ $\quad 1280 = 5r^4$

$256 = r^4$ $\quad \boxed{r = 4}$

28. Find the sum of the geometric series $2, \overset{x-2}{-}4, 8, -16, \ldots$

infinite geometric series

$|r| > 1$

$|-2| = 2 > 1$

Thus, the series diverges and has no definable sum.

29. Find the sum of all numbers between 50 and 685 that are multiples of 14.

Finite Arithmetic Series

$56, 70, \ldots, 672$

$672 = 56 + 14(n-1)$

$616 = 14(n-1)$

$44 = n-1 \quad n = 45$

$S_{45} = \dfrac{45(56+672)}{2} = \boxed{16,380}$

30. Evaluate $\lim\limits_{n\to\infty} \log_2\left(\dfrac{8n-8}{n+2}\right)$.

$\lim\limits_{n\to\infty} \log_2(8) = \boxed{3}$

31. Evaluate $\lim\limits_{n\to\infty} \dfrac{-2x^4 + 8x^3 - 7x}{4x^3 - x} = \lim\limits_{n\to\infty} -2x + 2 = -\infty$

*Given the infinite geometric series $400 \overset{x^{-1/4}}{-} 100 \overset{x^{-1/4}}{+} 25 - 6.25 + \ldots$

32. Find the sum.

$|{-1/4}| < 1$ ✓

$S = \dfrac{t_1}{1-r} = \dfrac{400}{1-(-1/4)} = \dfrac{400}{5/4} = \boxed{320}$

33. Rewrite using sigma notation.

$\sum\limits_{k=1}^{\infty} 400\left(\frac{-1}{4}\right)^{k-1}$

34. Given the infinite geometric series $\dfrac{x}{2} + \dfrac{x^2}{4} + \dfrac{x^3}{8} + \dfrac{x^4}{16} \ldots$, what is the interval of convergence?

$r = \dfrac{x}{2}$ $\quad \left|\dfrac{x}{2}\right| < 1$ $\quad \dfrac{x}{2} < 1$ $\quad \dfrac{x}{2} > -1$ $\quad \boxed{-2 < x < 2}$

$x < 2$ and $x > -2$

35. Write the series $\sum\limits_{k=1}^{6}(4+5k)$ in expanded form.

$4 + 5(1) + 4 + 5(2) + 4 + 5(3) + 4 + 5(4) + 4 + 5(5) + 4 + 5(6)$

$9 + 14 + 19 + 24 + 29 + 34$

36. Express the series $2 + 6 + 12 + 20 + \ldots$ using sigma notation.

$t_1 \quad t_2 \quad t_3 \quad t_4$

$\sum\limits_{k=1}^{\infty} k(k+1)$

Pre-Calculus
1st Semester Exam Review

*For the function $y=\frac{x^2-4}{x-2}$: $\frac{(x+2)(x-2)}{(x-2)}$

37. There is/are vertical asymptote(s) at:

none

38. There is a horizontal asymptote at:

none $\lim_{x\to\pm\infty} \frac{x^2-4}{x-2} = x$ $\lim_{x\to\infty} f(x) = \infty$ $\lim_{x\to-\infty} f(x) = -\infty$

39. There is/are zero(es) at:

$x=-2$

40. There is/are hole(s) at:

$x=2$

41. $\lim_{x\to\infty}\frac{x^2-4}{x-2}=$ $\lim_{x\to\infty} x = \infty$

42. Consider the function $f(x)=\frac{2}{3}x^3+2x-1$ and the point $(3,23)$ on its graph. What is the slope of the line tangent to $f(x)$ at $(3,23)$?

$f'(x) = 2x^2+2$ slope

$f'(3) = 2(3)^2+2 = \boxed{20}$

43. Consider the function $f(x)=x^4-2x^3+6$. At what point(s) will the tangent line be horizontal?

$f'(x) = 4x^3-6x^2$ $0 = 2x^2(2x-3)$ $(0,6)$

$0 = 4x^3-6x^2$ $x=0$ $x=3/2$ $(3/2, 69/16)$

44. Given $h(x)=\frac{-2}{\sqrt{x}}$, what is $h'(x)$?

$h(x) = -2x^{-1/2}$ $h'(x) = x^{-3/2} = \frac{1}{\sqrt[2]{x^3}} = \frac{\sqrt{x}}{x^2}$

45. Write out an expression using limits to find the slope of $y=(x+1)^2$.

$\lim_{h\to 0} \frac{(x+h+1)^2-(x+1)^2}{h}$

46. Find y''' given that $y=(x+5)(2x^2-1)$. $= 2x^3+10x^2-x-5$

$y' = 6x^2+20x-1$

$y'' = 12x+20$

$y''' = 12$

47. $h(x)$ and $m(x)$ are two functions defined in terms of x. Given that $h(2)=4$, $h'(2)=-1$, $m(2)=-3$, and $m'(2)=\frac{1}{2}$, calculate $[h(x)m(x)]'$ when $x=2$.

$mh' + m'h$

$-3(-1) + \frac{1}{2}(4)$

$3+2 = \boxed{5}$