Name: Key Date:________

Intro to Calculus
Chapter 3.4 to 3.6 Review

1. What is the instantaneous rate of change at $x=-1$ of the function f given by $f(x)=\dfrac{3x-2}{-2x^3+5x}$?

Instantaneous rate of change: $\boxed{f'(-1)=\dfrac{-14}{9}}$

$$f'(x)=\frac{(-2x^3+5x)(3)-(3x-2)(-6x^2+5)}{(-2x^3+5x)^2}$$

$$f'(x)=\frac{-6x^3+\cancel{15x}+18x^3-12x^2-\cancel{15x}+10}{(-2x^3+5x)^2}$$

$$f'(x)=\frac{12x^3-12x^2+10}{(-2x^3+5x)^2}$$

$$f'(-1)=\frac{12(-1)^3-12(-1)^2+10}{(-2(-1)^3+5(-1))^2}$$

$$f'(-1)=-14/9$$

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2. A particle starts at time $t=0$ and moves along the x-axis so that its position at any time $t\geq 0$ is given by $x(t)=t^3-12t+5$.

(a) Find the velocity of the particle at any time $t\geq 0$.

$V(t)=3t^2-12$

(b) Find the acceleration and jerk of the particle at any time $t\geq 0$.

$a(t)=6t$ $\quad$ $j(t)=6$

(c) Find all values of t for which the particle is at rest.

$V(t)=0=3t^2-12$ $\quad$ $12=3t^2$ $\quad$ $4=t^2$

$t=\pm 2$

$\boxed{\text{when } t\geq 0,\ t=2 \text{ seconds}}$

(d) Find the speed of the particle when its acceleration is zero.

$6t=0$
$t=0$

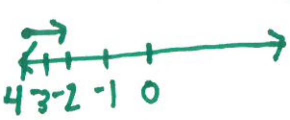
$|V(0)|=|-12|=\boxed{12}$

(e) Is the particle moving toward the origin ~~(negative direction)~~ or away from the origin ~~(positive direction)~~ when $t=3$?

$S(3)=(3)^3-12(3)+5$

$x(3)=27-36+5=-4$ position

$\boxed{\text{Toward the origin}}$



$V(3)=3(3)^2-12=27-12=15$ positive (to the right)

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3. If $f(x)=-2x+3x^2+\dfrac{1}{3x^2}$, then $f'(5)=$?

$$f'(5)=-2+6(5)\frac{-2}{3(5)^3}=-2+30-\frac{2}{375}$$

$$f'(x)=-2+6x\,\frac{-2}{3x^3}$$

$$\boxed{f'(5)=\frac{10498}{375}}$$

4. A particle starts at time $t = 0$ and moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = 2(t-4)^2 - 4t$.

(a) Find the velocity of the particle at any time $t \geq 0$

$V(t) = 4(t-4) - 4 = 4t - 20$

$\boxed{V(t) = 4t - 20}$

(b) Find the value of t when the particle is not moving but has a positive acceleration.

$V(t) = 0 = 4t - 20$ $\quad$ $a(t) = 4$

$20 = 4t$ $\quad$ $\boxed{5 \text{ seconds}}$

$5 = t$

(c) For what values of t is the velocity of the particle less than zero?

$\boxed{0 \leq t < 5 \text{ seconds}}$

(d) Find the acceleration at $t = 2$ seconds. Is the particle speeding up or slowing down?

$a(2) = 4$ $\quad$ since velocity is negative and acceleration is positive

$V(2) = -12$ $\quad$ The particle is $\boxed{\text{slowing down}}$

(e) At $t = 6$ seconds, is the particle speeding up or slowing down?

$a(6) = 4$ $\quad$ Since both velocity & acceleration are positive,

$V(6) = 4$ $\quad$ the particle is $\boxed{\text{speeding up}}$

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5. Write the equation of the line tangent to the graph of $f(x) = \frac{32x^5}{5} - x$ at the point where $f'(x) = 1$. $= m$

$f'(x) = 32x^4 - 1 = 1$ $\quad$ $f(\frac{1}{2}) = \frac{1}{5} - \frac{1}{2} = \frac{-3}{10}$ $\quad$ $\frac{-3}{10} = 1(\frac{1}{2}) + b$

$32x^4 = 2$ $\quad$ $\frac{-4}{5} = b$

$x^4 = \frac{1}{16}$ $\quad$ tangent line at $x = \frac{1}{2}$

$x = \frac{1}{2}$ $\quad$ $\boxed{y = x - \frac{4}{5}}$

6. A particle starts at time $t = 0$ and moves along the line ~~$x = 2$~~ x-axis so that its position at any time $t \geq 0$ is given by $x(t) = 3t(t+3)(t-1)$. $= (3t^2 + 9t)(t-1) = 3t^3 + 9t^2 - 3t^2 - 9t = 3t^3 + 6t^2 - 9t$

(a) When is the particle at rest? (round to the nearest thousand)

$V = 9t^2 + 12t - 9 = 0$

$3(3t^2 + 4t - 3) = 0$

$x = \frac{-12 \pm \sqrt{(144) - 4(9)(-9)}}{18}$

$t = \frac{-12 \pm \sqrt{468}}{18}$

$t \approx 0.535, \cancel{-1.369}$

$\boxed{t \approx 0.535 \text{ seconds}}$

~~(b) What is the highest position the particle reaches?~~

~~(c) What is the total distance the particle has traveled after 5 seconds?~~

~~(d) Is the particle's speed increasing or decreasing at $t = 4$ seconds?~~

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7. If $f(x) = \dfrac{2x^2 + 3}{x - 7 + 4x^4}$, then $f'(2) = ?$

$\boxed{f'(2) = \dfrac{-947}{3481}}$

$f'(2) \approx -0.272$

$$f'(x) = \frac{(x-7+4x^4)(4x) - (2x^2+3)(1+16x^3)}{(x-7+4x^4)^2}$$

$$f'(2) = \frac{(2-7+4(2)^4)(8) - (2(2)^2+3)(1+16(2)^3)}{(2-7+4(2)^4)^2} = \frac{59(8) - 11(129)}{3481}$$

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8. A particle moves along a straight line with velocity given by $v(t) = \cancel{2.168e^{2t+1}}$ $3t^3 - 4t^2 + 5$ at time $t \geq 0$. What is the acceleration of the particle at time $t = 2$?

$a(t) = 9t^2 - 8t$

$a(2) = 9(2)^2 - 8(2) = \boxed{20 \text{ meters/sec}^3}$

9. For $f(x) = (2x+7)(6x^4 - 3x^3)^4$, find $f'(x)$. $f'(x) = 2(6x^4-3x^3)^4 + (2x+7)4(6x^4-3x^3)^3(24x^3-9x^2)$

$\boxed{f'(x) = 2(6x^4-3x^3)^4 + (8x+28)(6x^4-3x^3)^3(24x^3-9x^2)}$

10. For $g(x) = \tan(2x + \pi)$, find $g'\left(\dfrac{\pi}{2}\right)$.

$\boxed{g'(x) = 2\sec^2(2x+\pi)}$

$g'\left(\frac{\pi}{2}\right) = 2\sec^2\left(2\left(\frac{\pi}{2}\right) + \pi\right)$

$g'\left(\frac{\pi}{2}\right) = 2\sec^2(2\pi) = \boxed{2}$

1. Multiple Choice Which of the following gives dy/dx for $y = \sin^4(3x)$?

(A) $4 \sin^3(3x) \cos(3x)$

(B) $12 \sin^3(3x) \cos(3x)$

(C) $12 \sin(3x) \cos(3x)$

(D) $12 \sin^3(3x)$

(E) $-12 \sin^3(3x) \cos(3x)$

2. Multiple Choice Which of the following gives y'' for $y = \cos x + \tan x$?

(A) $-\cos x + 2\sec^2 x \tan x$ (B) $\cos x + 2\sec^2 x \tan x$

(C) $-\sin x + \sec^2 x$ (D) $-\cos x + \sec^2 x \tan x$

(E) $\cos x + \sec^2 x \tan x$

72. Multiple Choice Which of the following is dy/dx if $y = \tan(4x)$?

(A) $4\sec(4x)\tan(4x)$ (B) $\sec(4x)\tan(4x)$ (C) $4\cot(4x)$

(D) $\sec^2(4x)$ (E) $4\sec^2(4x)$

73. Multiple Choice Which of the following is dy/dx if $y = \cos^2(x^3 + x^2)$?

(A) $-2(3x^2 + 2x)$

(B) $-(3x^2 + 2x)\cos(x^3 + x^2)\sin(x^3 + x^2)$

(C) $-2(3x^2 + 2x)\cos(x^3 + x^2)\sin(x^3 + x^2)$

(D) $2(3x^2 + 2x)\cos(x^3 + x^2)\sin(x^3 + x^2)$

(E) $2(3x^2 + 2x)$

Differentiate the following:

21. $y = \sin^2(3x - 2)$ **22.** $y = (1 + \cos 2x)^2$

23. $y = (1 + \cos^2 7x)^3$ **24.** $y = \sqrt{\tan 5x}$

16. $y = x^3(2x - 5)^4$

21. $y' = 6\sin(3x-2)\cos(3x-2)$ 22. $y' = -4(1+\cos 2x)\sin 2x$

23. $y' = -42(1+\cos^2 7x)^2 \cos 7x \cdot \sin 7x$ 24. $y' = \frac{5}{2}(\tan 5x)^{-1/2}\sec^2 5x$

16. $y' = 8x^3(2x-5)^3 + 3x^2(2x-5)^4$