

Implicit Differentiation

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y .

1) $2x^3 = 2y^2 + 5$

2) $3x^2 + 3y^2 = 2$

3) $5y^2 = 2x^3 - 5y$

4) $4x^2 = 2y^3 + 4y$

5) $5x^3 = -3xy + 2$

6) $1 = 3x + 2x^2y^2$

7) $3x^2y^2 = 4x^2 - 4xy$

8) $5x^3 + xy^2 = 5x^3y^3$

9) $2x^3 = (3xy + 1)^2$

10) $x^2 = (4x^2y^3 + 1)^2$

11) $\sin 2x^2y^3 = 3x^3 + 1$

12) $3x^2 + 3 = 5xy^2$

For each problem, use implicit differentiation to find $\frac{d^2y}{dx^2}$ in terms of x and y .

13) $4y^2 + 2 = 3x^2$

14) $5 = 4x^2 + 5y^2$

Critical thinking question:

- 15) Use three strategies to find $\frac{dy}{dx}$ in terms of x and y , where $\frac{3x^2}{4y} = x$. Strategy 1: Use implicit differentiation directly on the given equation. Strategy 2: Multiply both sides of the given equation by the denominator of the left side, then use implicit differentiation. Strategy 3: Solve for y , then differentiate. Do your three answers look the same? If not, how can you show that they are all correct answers?

Implicit Differentiation

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y .

1) $2x^3 = 2y^2 + 5$

$$\frac{dy}{dx} = \frac{3x^2}{2y}$$

2) $3x^2 + 3y^2 = 2$

$$\frac{dy}{dx} = -\frac{x}{y}$$

3) $5y^2 = 2x^3 - 5y$

$$\frac{dy}{dx} = \frac{6x^2}{10y + 5}$$

4) $4x^2 = 2y^3 + 4y$

$$\frac{dy}{dx} = \frac{4x}{3y^2 + 2}$$

5) $5x^3 = -3xy + 2$

$$\frac{dy}{dx} = \frac{-y - 5x^2}{x}$$

6) $1 = 3x + 2x^2y^2$

$$\frac{dy}{dx} = \frac{-3 - 4xy^2}{4x^2y}$$

7) $3x^2y^2 = 4x^2 - 4xy$

$$\frac{dy}{dx} = \frac{4x - 2y - 3xy^2}{3x^2y + 2x}$$

8) $5x^3 + xy^2 = 5x^3y^3$

$$\frac{dy}{dx} = \frac{15x^2y^3 - 15x^2 - y^2}{2xy - 15x^3y^2}$$

9) $2x^3 = (3xy + 1)^2$

$$\frac{dy}{dx} = \frac{-3y^2x - y + x^2}{3x^2y + x}$$

10) $x^2 = (4x^2y^3 + 1)^2$

$$\frac{dy}{dx} = \frac{-32y^6x^2 - 8y^3 + 1}{48x^3y^5 + 12xy^2}$$

11) $\sin 2x^2y^3 = 3x^3 + 1$

$$\frac{dy}{dx} = \frac{9x - 4y^3 \cos 2x^2y^3}{6xy^2 \cos 2x^2y^3}$$

12) $3x^2 + 3 = 5xy^2$

$$\frac{dy}{dx} = \frac{6x - 5y^2}{10xy}$$

For each problem, use implicit differentiation to find $\frac{d^2y}{dx^2}$ in terms of x and y .

13) $4y^2 + 2 = 3x^2$

$$\frac{d^2y}{dx^2} = \frac{12y^2 - 9x^2}{16y^3}$$

14) $5 = 4x^2 + 5y^2$

$$\frac{d^2y}{dx^2} = \frac{-20y^2 - 16x^2}{25y^3}$$

Critical thinking question:

15) Use three strategies to find $\frac{dy}{dx}$ in terms of x and y , where $\frac{3x^2}{4y} = x$. Strategy 1: Use implicit differentiation directly on the given equation. Strategy 2: Multiply both sides of the given equation by the denominator of the left side, then use implicit differentiation. Strategy 3: Solve for y , then differentiate. Do your three answers look the same? If not, how can you show that they are all correct answers?

Strategy 1: $\frac{dy}{dx} = \frac{6xy - 4y^2}{3x^2}$, Strategy 2: $\frac{dy}{dx} = \frac{6x - 4y}{4x}$, Strategy 3: $\frac{dy}{dx} = \frac{3}{4}$ To show all

answers are the same, plug $y = \frac{3x}{4}$ into results for strategies 1 and 2.